Antenna Theory

1 Introduction

Antennas are device that designed to radiate electromagnetic energy efficiently in a prescribed manner. It is the **current distributions on the antennas that produce the radiation**. Usually this current distributions are excited by transmission lines and waveguides.
Propagation mode adapter

During both transmission and receive operations the antenna must provide the transition between these two propagation modes.
Transmitting Antenna Equivalent Circuit

- $V_g$ - voltage-source generator (transmitter);
- $Z_g$ - impedance of the generator (transmitter);
- $R_{rad}$ - radiation resistance, represents energy radiated into space (related to the radiated power as)
  \[ P_{rad} = I_A^2 \cdot R_{rad} \]
- $R_L$ - loss resistance, (related to conduction and dielectric losses); i.e. transformed into heat in the antenna structure
- $jX_A$ – antenna reactance.

Transmission-line Thevenin equivalent circuit of a radiating (transmitting) system. The antenna is represented by its input impedance (which is frequency-dependent and is influenced by objects nearby) as seen from the generator.

\[ Z_A = R_{rad} + R_L + jX_A \]
Transmission-line Thevenin equivalent circuit of a receiving antenna system.

Note: The antenna impedance is the same when the antenna is used to radiate and when it is used to receive energy.
Table 1-2  Frequencies and wavelengths of the electromagnetic spectrum from almost dc to gamma rays

<table>
<thead>
<tr>
<th>Frequency $f = \frac{c}{\lambda}$</th>
<th>Wavelength $\lambda = \frac{c}{f}$</th>
<th>Relevant dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \text{ Hz}$</td>
<td>$10^3 \text{ m}$</td>
<td>Earth diameter</td>
</tr>
<tr>
<td>$10^3 \text{ kHz}$</td>
<td>$10^6 \text{ m}$</td>
<td>Mt. Everest</td>
</tr>
<tr>
<td>$10^6 \text{ MHz}$</td>
<td>$10^3 \text{ m}$</td>
<td>Redwood tree</td>
</tr>
<tr>
<td>$10^9 \text{ GHz}$</td>
<td>$1 \text{ m}$</td>
<td>Human</td>
</tr>
<tr>
<td>$10^{12} \text{ THz}$</td>
<td>$10^{-3} \text{ m}$</td>
<td>Hydrogen line</td>
</tr>
<tr>
<td>$10^{-3} \text{ mm}$</td>
<td>$10^{-3} \text{ m}$</td>
<td>O$_2$ line</td>
</tr>
<tr>
<td>$10^{-6} \text{ nm}$</td>
<td>$10^{-3} \text{ m}$</td>
<td>Molecular lines</td>
</tr>
<tr>
<td>$10^{-9} \text{ pm}$</td>
<td>$10^{-3} \text{ m}$</td>
<td>Sand grain</td>
</tr>
<tr>
<td>$10^{-12} \text{ fm}$</td>
<td>$10^{-3} \text{ m}$</td>
<td>Bacterium</td>
</tr>
<tr>
<td>$10^{-15} \text{ am}$</td>
<td>$10^{-3} \text{ m}$</td>
<td>Virus</td>
</tr>
<tr>
<td>$10^{-18} \text{ pm}$</td>
<td>$10^{-3} \text{ m}$</td>
<td>Atomic spacing</td>
</tr>
<tr>
<td>$10^{-21} \text{ fm}$</td>
<td>$10^{-3} \text{ m}$</td>
<td>Atom</td>
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After Kraus & Marhefka, 2003
### Radio-frequency band names†

<table>
<thead>
<tr>
<th>Name</th>
<th>Frequency</th>
<th>Principal use</th>
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</thead>
<tbody>
<tr>
<td>ELF†</td>
<td>3–30 Hz</td>
<td>Power grids</td>
</tr>
<tr>
<td>SLF</td>
<td>30–300 Hz</td>
<td></td>
</tr>
<tr>
<td>ULF</td>
<td>300–3000 Hz</td>
<td></td>
</tr>
<tr>
<td>VLF</td>
<td>3–30 kHz</td>
<td>Submarines</td>
</tr>
<tr>
<td>LF</td>
<td>30–300 kHz</td>
<td>Beacons</td>
</tr>
<tr>
<td>MF</td>
<td>300–3000 kHz</td>
<td>AM broadcast</td>
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<tr>
<td>HF</td>
<td>3–30 MHz</td>
<td>Shortwave broadcast</td>
</tr>
<tr>
<td>VHF</td>
<td>30–300 MHz</td>
<td>FM, TV</td>
</tr>
<tr>
<td>UHF</td>
<td>300–3000 MHz</td>
<td>TV, LAN, cellular, GPS</td>
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<tr>
<td>SHF</td>
<td>3–30 GHz</td>
<td>Radar, GSO satellites, data</td>
</tr>
<tr>
<td>EHF</td>
<td>30–300 GHz</td>
<td>Radar, automotive, data</td>
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</tbody>
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#### Microwave bands

<table>
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<th>“New”</th>
<th>Frequency</th>
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</thead>
<tbody>
<tr>
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<td>D</td>
<td>1–2 GHz</td>
</tr>
<tr>
<td>S</td>
<td>E, F</td>
<td>2.4 GHz</td>
</tr>
<tr>
<td>C</td>
<td>G, H</td>
<td>4–8 GHz</td>
</tr>
<tr>
<td>X</td>
<td>I, J</td>
<td>8–12 GHz</td>
</tr>
<tr>
<td>Ku</td>
<td>J</td>
<td>12–18 GHz</td>
</tr>
<tr>
<td>K</td>
<td>J</td>
<td>18–26 GHz</td>
</tr>
<tr>
<td>Ka</td>
<td>K</td>
<td>26–40 GHz</td>
</tr>
</tbody>
</table>

†ELF = Extremely Low Frequency, SLF = Super-Low Frequency, VLF = Very Low Frequency, MF = Medium Frequency, HF = High Frequency, UHF = Ultrahigh Frequency, etc.
Theory

Antennas include wire and aperture types. Wire types include dipoles, monopoles, loops, rods, stubs, helicities, Yagi-Udas, spirals. Aperture types include horns, reflectors, parabolic, lenses.

![Antenna Types](image)

Fig. 3.1 Antenna types commonly used in microwave remote sensing.
Theory

In **wire-type antennas** the radiation characteristics are determined by the current distribution which produces the local magnetic field.

Yagi-Uda antenna

Helical antenna
Theory – wire antenna example

Consider a thin linear antenna of arbitrary length $l$, with no restrictions on $l$ compared with the wavelength $\lambda$. The antenna, shown in Fig. 3.9, is fed at its center with a sinusoidal current distribution,

$$I = I_0 \sin[k(l/2 - |z|)].$$

The far-zone fields from an infinitesimal dipole of length $dz$ at a distance $s$ are

$$dE_\theta = \frac{jk \eta I}{4\pi s} dz \ e^{-js\sin \theta}, \quad dH_\phi = \frac{1}{\eta} dE_\theta.$$

The far fields of the entire antenna now may be obtained by integrating the fields from all of the Hertzian dipoles making up the antenna:

$$E_\theta = \int_{-L/2}^{L/2} dE_\theta.$$

Some simplifying approximations can be made to take advantage the far-field conditions.

$$\theta_s \approx \theta \quad s = \sqrt{r^2 + z^2 - 2rz \cos \theta} \approx r - z \cos \theta$$

$$dE_\theta = \frac{jk \eta I}{4\pi r} dz \ e^{-jkr \sin \theta} e^{jkz \cos \theta}$$

The following expressions for the far-zone fields of the long linear antenna are obtained:

$$E_\theta = \frac{j \eta I_0}{2\pi r} \left[ \frac{\cos(\frac{1}{2}kl \cos \theta) - \cos(\frac{1}{2}kl)}{\sin \theta} \right] e^{-jkr}, \quad H_\phi = \frac{1}{\eta} E_\theta.$$
Theory – wire antenna example

Once $E_0$ and $E_\phi$ are known, the radiation characteristics can be determined. Defining the directional function $f(\theta, \phi)$ from

$$E_\theta = \frac{e^{-jkr}}{r} f_1(\theta, \phi) \quad E_\phi = \frac{e^{-jkr}}{r} f_2(\theta, \phi)$$

where $k = 2\pi/\lambda$. The power flow in the far field is then given by

$$S_r = \frac{1}{2\eta r^2} \left( |f_1(\theta, \phi)|^2 + |f_2(\theta, \phi)|^2 \right).$$

Instead of using the power density $S_r(r, \theta, \phi)$ to describe the directional properties of an antenna, it is usually more convenient to use an $r$-independent function known as the radiation intensity, or radiation pattern, $F(\theta, \phi)$. This function is given by

$$F(\theta, \phi) = r^2 S_r = \frac{1}{2\eta} \left( |f_1(\theta, \phi)|^2 + |f_2(\theta, \phi)|^2 \right)$$

where $F(\theta, \phi)$ is now expressed in watts per unit solid angle (watts per steradian). It is customary to normalize the maximum value of $F(\theta, \phi)$ to unity, in which case the pattern is referred to as the normalized radiation pattern, $F_n(\theta, \phi)$. Thus,

$$F_n(\theta, \phi) = \frac{F(\theta, \phi)}{F(\theta, \phi)_{\text{max}}} = \frac{S_r(r, \theta, \phi)}{S_r(r, \theta, \phi)_{\text{max}}},$$

with the understanding that $r$ is held constant.
What makes a short dipole?

Length $l$ is very short compared to wavelength ($l<<\lambda$, i.e. $l$ should not exceed $\lambda/50$) which is also called Hertzian dipole.

Carries uniform current $I$ along the entire length $l$. To allow such uniform current, **we attach plates** at the ends of the dipole as capacitive load. However, we assume the plates are small that their radiation is negligible.

The dipole may be energized by **balanced transmission line**. However, it is assumed that the transmission line does not radiate.

The diameter $d$ of the dipole is small compared to its length ($d<<L$, ).

Thus a **short dipole** consist a simple of a thin conductor of length $L$ with a uniform current $I$ and point charges $q$ at the ends.

$$\frac{dq}{dt} = I$$
Wire antenna (half wave dipole)

The most basic form of antenna, and most popular

Transmission line

Total length of the radiating element is half-wavelength

Radiation pattern of a $\lambda/2$ thin wire dipole – omni-directional

E-plane

H-plane
A quarter-wave monopole antenna excited by a source at its base exhibits the same radiation pattern in the region above the ground plane as a half-wave dipole in free space.

- Use image theory for analysis.
- Hence, a monopole radiates only half as much power as the dipole.
Monopole antenna

Mirroring principle creates image of monopole, transforming it into a dipole.

Radiation pattern of vertical monopole above ground of (A) perfect and (B) average conductivity.
Ground plane

A ground plane will produce an image of nearby currents. The image will have a phase shift of $180^\circ$ with respect to the original current. Therefore as the current element is placed close to the surface, the induced image current will effectively cancel the radiating fields from the current.

The ground plane may be any conducting surface including a metal sheet, a water surface, or the ground (soil, pavement, rock).
Dipole of other lengths

(a) $l = \lambda/2$
(b) $l = \lambda$
(c) $l = 3\lambda/2$

(a) $l = \lambda/2$

(b) $l = \lambda$

(c) $l = 3\lambda/2$
Field regions

Radiating near field (Fresnel) region

Reactive near field region

No abrupt changes in the field configurations are noted as the boundaries are crossed – but there are distinct differences between the fields.

Far field (Fraunhofer) region
Field Regions

Far-field (Fraunhofer) region

Radiating near-field (Fresnel) region

Reactive near-field region

\[ R_1 = 0.62 \sqrt{D^3/\lambda} \]

\[ R_2 = 2D^2/\lambda \]

Fig. 2.7
Energy is transferred to and from the near field region which represents the reactive part of the antenna driving point impedance.

As one moves further away, this oscillatory energy flow reduces leaving just the regular power flow in the resistive characteristic impedance (377 ohms or 120 pi ohms) of free space.

In the far field the polar radiation pattern is completely independent of distance from the radiating source.
Evolution of Pattern from Near to Far Field

- Reactive Near-field
- Radiating Near-field
- Field Distribution
- Fresnel
- Far-Field

Fig. 2.8
Reactive near field region

“That portion of the near field region immediately surrounding the antenna wherein the reactive field predominates”

Outer boundary \( R < 0.62\sqrt{D^3/\lambda} \) 

For short dipole boundary is \( \lambda / 2\pi \)

\[
H_\phi = \frac{I_m L \sin \theta e^{-j\beta r}}{4\pi r^2}
\]

\[
E_r = \frac{I_m L \cos(\theta) e^{-j\beta}}{j2\pi \omega \varepsilon r^3}
\]

\[
E_\theta = \frac{I_m L \sin(\theta) e^{-j\beta}}{j4\pi \omega \varepsilon r^3}
\]

Fields of a short dipole can be approximated by these expressions.

The E field components are in time phase, but they are in time phase quadrature with the H field. Thus, there is no time average power flow.
Radiating near field region

“That region of the field of an antenna between the reactive near field region and the far field region wherein radiation fields predominate and wherein the angular field distribution is dependant upon the distance from the antenna”

If the antenna has a maximum dimension that is not large compared to the wavelength, this region may not exist.

\[ R < \frac{2D^2}{\lambda} \]
\[ R < 0.62\sqrt{\frac{D^3}{\lambda}} \]

Inner boundary \hspace{2cm} Outer boundary
Antenna Terminology
Radiation intensity

- **Radiation intensity**:- In a given direction, the power radiated from an antenna per unit solid angle
- It is a far field parameter, and can be obtained by multiplying radiation density (magnitude of Poynting vector) with the square of distance
- Its denoted by U
- Unit is Watts per steradian (W/sr)
2.2 Power Intensity

\[ U = r^2 |P_{\text{av}}| \text{ (W/sr)} \]

sr = steradian, unit for measuring the solid angle.

Solid angle \( \Omega \) is the ratio of that part of a spherical surface area \( S \) subtended at the centre of a sphere to the square of the radius of the sphere.

\[ \Omega = \frac{S}{r^2} \text{ (sr)} \]

The solid angle subtended by a whole spherical surface is therefore:

\[ \Omega = \frac{4\pi r^2}{r^2} = 4\pi \text{ (sr)} \]

Note that \( U \) is a function of direction \((\theta, \phi)\) only and not distance \((r)\).
Beam solid angle (beam area)

What is solid angle?

It’s like the angle in 3D, one sphere has $4\pi$ solid angle.

Beam solid angle - The solid angle through which all the power would be radiated if the power per unit solid angle (radiation intensity) equals the maximum value over the beam area $\Omega_A$. 

"power per unit solid angle"
2.3 Radiated Power

\[ P_{\text{rad}} = \iint_{s} P_{av} \cdot ds = \frac{1}{2} \iint_{s} \text{Re}[E \times H^*] \cdot ds \quad (W) \]

Note that the integration is over a closed surface with the antenna inside and the surface is sufficiently far from the antenna (far field conditions).
 Beamwidth and beam solid angle

The beam or pattern solid angle, \( \Omega_p \) [steradians or sr] is defined as

\[
\Omega_p = \int \int F_n(\theta, \phi) d\Omega
\]

where \( d\Omega \) is the elemental solid angle given by

\[
d\Omega = \sin \theta \, d\theta \, d\phi
\]

**Fig. 3.5** The solid angle of a unidirectional radiation pattern is approximately equal to the product of the half-power beamwidths in the two principal planes, i.e., \( \Omega_p = \beta_{xz} \beta_{yz} \).
Antenna efficiency

The radiation efficiency $\eta$ indicates how efficiently the antenna uses the RF power. The efficiency of an antenna is defined as the ratio of the radiated power to the total input power supplied to the antenna and is given by:

$$\eta = \frac{\text{Radiated power}}{\text{Total input power}} = \frac{G_p}{G_d} = \frac{P_t}{P_t + P_l}$$

$$\eta \% \approx \frac{R_r}{R_r + R_l} \times 100$$

$R_r$ is the radiation resistance and $R_l$ is the Ohmic loss resistance of the antenna conductor.
Directivity

• The ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions
• Tells us how well the antenna is radiating towards a particular direction
• For an isotropic antenna, the directivity is equal to unity
• Does not take into account the efficiency of the antenna
Gain

• The ratio of the radiation intensity, in a given direction, to the radiation intensity that would be obtained if the power accepted by the antenna were radiated isotropically.

• Gain does not include losses arising from impedance and polarization mismatches.

• If there is no loss in the antenna, gain equals directivity.

\[ G = \eta D \]
Directivity, gain, effective area

**Directivity** – the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

\[
D(\theta, \phi) = \frac{F_n(\theta, \phi)}{\frac{1}{4\pi} \int \int F_n(\theta, \phi) \, d\Omega} \quad \text{[unitless]}
\]

Maximum directivity, \(D_o\), found in the direction \((\theta, \phi)\) where \(F_n = 1\)

\[
D_o = \frac{4\pi}{\int \int F_n(\theta, \phi) \, d\Omega} = \frac{4\pi}{\Omega_p} \quad \text{and} \quad \Omega_p \approx \beta_{xz} \beta_{yz} \quad \text{or} \quad D_0 = \frac{4\pi}{\Omega_p} \approx \frac{4\pi}{\beta_{xz} \beta_{yz}}
\]

Given \(D_o\), \(D\) can be found

\[
D(\theta, \phi) = D_0 F_n(\theta, \phi)
\]
Directivity, gain, effective area

**Gain** – ratio of the power at the input of a loss-free isotropic antenna to the power supplied to the input of the given antenna to produce, in a given direction, the same field strength at the same distance.

**Efficiency** – Of the total power $P_t$ supplied to the antenna, a part $P_o$ is radiated out into space and the remainder $P_l$ is dissipated as heat in the antenna structure. The *radiation efficiency* $\eta_l$ is defined as the ratio of $P_o$ to $P_t$

$$\eta_l = \frac{P_o}{P_t}$$

Therefore gain, $G$, is related to directivity, $D$, as

$$G(\theta, \phi) = \eta_l \ D(\theta, \phi)$$

And maximum gain, $G_o$, is related to maximum directivity, $D_o$, as

$$G_o = \eta_l \ D_o$$
Directivity, gain, effective area

Effective area – the functional equivalent area from which an antenna directed toward the source of the received signal gathers or absorbs the energy of an incident electromagnetic wave

It can be shown that the maximum directivity \( D_0 \) of an antenna is related to an effective area (or effective aperture) \( A_{\text{eff}} \), by

\[
D_0 = \frac{4 \pi}{\lambda^2} A_{\text{eff}} = \frac{4 \pi}{\lambda^2} \eta_a A_p
\]

where \( A_p \) is the physical aperture of the antenna and \( \eta_a = A_{\text{eff}} / A_p \) is the aperture efficiency \((0 \leq \eta_a \leq 1)\). Consequently

\[
A_{\text{eff}} = \frac{\lambda^2}{\Omega_p} \simeq \frac{\lambda^2}{\beta_{xz} \beta_{yz}} \quad [\text{m}^2]
\]

For a rectangular aperture with dimensions \( l_x \) and \( l_y \) in the x- and y-axes, and an aperture efficiency \( \eta_a = 1 \), we get

\[
\beta_{xz} \simeq \frac{\lambda}{l_x} \quad [\text{rad}] \quad \beta_{yz} \simeq \frac{\lambda}{l_y} \quad [\text{rad}]
\]
Directivity, gain, effective area

Therefore the maximum gain and the effective area can be used interchangeably by assuming a value for the radiation efficiency (e.g., \( \eta_l = 1 \))

\[
G_0 = \frac{4\pi}{\lambda^2} \eta_l A_{\text{eff}}
\]

\[
G_0 \approx A_{\text{eff}} \frac{4\pi}{\lambda^2} = \frac{4\pi}{\beta_{xz} \beta_{yz}}
\]

\[
A_{\text{eff}} \approx G_0 \frac{\lambda^2}{4\pi}
\]

**Example:** For a 30-cm x 10-cm aperture, \( f = 10 \text{ GHz} \) (\( \lambda = 3 \text{ cm} \))

\( \beta_{xz} \approx 0.1 \text{ radian or } 5.7^\circ, \beta_{yz} \approx 0.3 \text{ radian or } 17.2^\circ \)

\( G_0 \approx 419 \) or 26 dBi

(dBi: dB relative to an isotropic radiator)
Bandwidth

The antenna’s bandwidth is the range of operating frequencies over which the antenna meets the operational requirements, including:

- Spatial properties (radiation characteristics)
- Polarization properties
- Impedance properties
- Propagation mode properties

Normally expressed as a fraction of centre frequency.

Most antenna technologies can support operation over a frequency range that is 5 to 10% of the central frequency

(e.g., 100 MHz bandwidth at 2 GHz)

$$\text{Impedance bandwidth} = \frac{f_U - f_L}{f_C} \times 100\%$$
Bandwidth

• The range of frequencies within which the performance of the antenna, with respect to some characteristic, conforms to a specified standard.
• Normally used standards - Impedance bandwidth; Gain bandwidth; Radiation pattern bandwidth; side lobe level; beamwidth; polarisation; beam direction.
Antenna pattern

• Also called as radiation pattern
• Defined as “the spatial distribution of a quantity that characterises the electromagnetic field generated by an antenna”.
• The quantities that are most often used are power flux density, radiation intensity, directivity, phase, polarizations and field strength.
E-plane and H-plane

- Is defined only for single linear polarised antenna
- The radiation pattern that contains the electric field is called the E-plane cut or pattern
- Automatically the other plane, which contains the Magnetic field is called the H-plane

For the short dipole, the left pattern cut is the E-plane and the right is H-plane

E- and H-plane does not mean anything for a dual polarised or circularly polarised antenna!
2-D Pattern

- Usually the antenna pattern is presented as a 2-D plot, with only one of the direction angles, $\theta$ or $\phi$ varies.
- It is an intersection of the 3-D one with a given plane:
  - usually it is a $\theta = const$ plane or a $\phi = const$ plane that contains the pattern’s maximum.
Isotropic and Omni-directional radiator

- **Isotropic**: A hypothetical, lossless antenna having equal radiation intensity in all directions.

\[
P_r = \frac{P_t}{4\pi r^2}
\]

For an isotropic radiator, the power density is given by dividing the total radiated power equally over the surface of the sphere.

- **Omni-directional**: An antenna having an essentially non-directional pattern in a given plane of the antenna and a directional pattern in any orthogonal plane.

  A typical example is the wire dipole (short dipole) – non directional in XY plane.
Major lobe & minor lobe

• Major lobe is also called main lobe
• Defined as “the radiation lobe containing the direction of maximum radiation”
• In certain antennas, such as multi-lobed or split beam antennas, there may exist more than one major lobe
• Minor lobe - A radiation lobe in any direction other than that of the major lobe
• When its adjacent to the main lobe its called side lobe
• Side lobe level – maximum relative directivity of the highest side lobe with respect to the maximum directivity of the antenna
• Back lobe – refers to a minor lobe that occupies the hemispheres in a direction opposite to that of the major lobe.
Radiation pattern lobes

(a) Polar Diagram

(b) Rectangular Plot
Pattern Lobes

- **Pattern lobe** is a portion of the radiation pattern with a local maximum
  - Lobes are classified as: major, minor, side lobes, back lobes.
Pattern Lobes and Beamwidths

Half-power beamwidth (HPBW)
First null beamwidth (FNBW)

Major lobe
Side lobe
Back lobe

Minor lobes

Radiation intensity

(b)

θ
Beamwidth - half-power beamwidth and first null beamwidth

• The width of the main beam or major lobe in terms of angles or radians.
• Half-power beamwidth (also known as 3dB beamwidth) and first null beamwidth is of interest
• 3dB beamwidth - In a radiation pattern cut containing the direction of the maximum of a lobe, the angle between the two directions in which the radiation intensity is one-half the maximum value
• Normally related to the resolution
• Narrow beam requires large antenna dimensions
• First-null beamwidth – In a radiation pattern cut, the angle between the two nulls adjacent to the main beam
Beamwidth

- **Half-power beamwidth** (HPBW) is the angle between two vectors from the pattern’s origin to the points of the major lobe where the radiation intensity is half its maximum.
  - Often used to describe the antenna resolution properties.
    - Important in radar technology, radioastronomy, etc.
- **First-null beamwidth** (FNBW) is the angle between two vectors, originating at the pattern’s origin and tangent to the main beam at its base.
  - Often FNBW ≈ 2*HPBW
Radiation Resistance

- An antenna’s radiation resistance is a measure of its ability to radiate an applied signal into space, or to receive a signal from space.
- The radiation resistance is not a real (dissipative) resistance, but a measure of the power radiated into free-space for a given input current.
- The important observation about radiation resistance is that, for a given current into the antenna, as radiation resistance increases, so does the antenna’s efficiency.
Radiation resistance

Radiation resistance $R_r$ of an antenna is the hypothetical resistance that would dissipate the same amount of power as the radiated power $R_r$.

Let's find the radiation resistance for a short dipole:

$$P_{rad} = \int P_{ave} \cdot dS$$

$$= \int \int \frac{\eta I_m^2 L^2 \beta^2}{32\pi^2 r^2} \sin^2 \theta \quad r^2 \sin \theta d\theta d\phi$$

First find the total power radiated

$$= \frac{\eta I_m^2 L^2 \beta^2}{32\pi^2} 2\pi \int_{0}^{\pi} \sin^3 \theta d\theta$$

$$= \frac{\eta I_m^2 L^2 \beta^2}{12\pi^2} \pi$$
Replace $\beta$ and $\eta$,

$$P_{rad} = 40\pi^2 \left[ \frac{L}{\lambda} \right]^2 I_m^2$$

The power is equivalent to the power dissipated in a fictitious resistance $R_r$ by a current $I_m$

$$P_{rad} = \frac{1}{2} I_m^2 R_r = 40\pi^2 \left[ \frac{L}{\lambda} \right]^2 I_m^2$$

Thus, the radiation resistance is given by,

$$R_r = 80\pi^2 \left[ \frac{L}{\lambda} \right]^2$$
2.7 Reflection Coefficient

The reflection coefficient of a transmitting antenna is defined by:

$$\rho = \frac{Z_A - Z_0}{Z_A + Z_0} \quad (\text{dimensionless})$$

$$\rho$$ can be calculated (as above) or measured. The magnitude of $$\rho$$ is from 0 to 1. When the transmitting antenna is not match, i.e., $$Z_A \neq Z_0$$, there is a loss due to reflection (return loss) of the wave at the antenna terminals. When expressed in dB, $$\rho$$ is always a negative number. Sometimes we use $$S_{11}$$ to represent $$\rho$. 
2.8 Return Loss

The return loss of a transmitting antenna is defined by:

\[
\text{return loss} = -20 \log |\rho| \quad \text{(dB)}
\]

Possible values of return loss are from 0 dB to \(\infty\) dB. Return loss is always a positive number.
Standing Wave ratio: The ratio of maximum to minimum voltages along a finite terminated line is called standing wave ratio.

\[ SWR = \left| \frac{V_{\text{max}}}{V_{\text{min}}} \right| = \frac{1+\Gamma_L}{1-\Gamma_L} \]

\[ \therefore \Gamma_L = \text{reflection coefficient is} \]

\[ = \frac{SWR - 1}{SWR + 1} \]

\[ = \frac{1.5 - 1}{1.5 + 1} = \frac{0.5}{1.5} = \frac{1}{3}. \]
2.9 VSWR

The voltage standing wave ratio (VSWR) of a transmitting antenna is defined by:

\[
VSWR = \frac{1 + |\rho|}{1 - |\rho|} \quad \text{(dimensionless)}
\]

Same as \( \rho \) and the return loss, VSWR is also a common parameter used to characterize the matching property of a transmitting antenna. Possible values of VSWR are from 1 to \( \infty \). \( VSWR = 1 \Rightarrow \) perfectly matched. \( VSWR = \infty \Rightarrow \) completely unmatched.
2.10 Impedance Bandwidth

\[ \rho \text{ or } S_{11} \text{ (dB)} \]

-10dB

\[ f_L \quad f_C \quad f_U \]

Impedance bandwidth

Frequency
Impedance bandwidth = $\frac{f_U - f_L}{f_C} \times 100\%$

Note that when $\rho = -10$ dB,

Prob:
What will be the VSWR if the reflection coefficient is -10dB
Impedance bandwidth = \( \frac{f_u - f_L}{f_c} \times 100\% \)

Note that when \( \rho = -10 \text{ dB} \),

\[
VSWR = \frac{1 + |\rho|}{1 - |\rho|} = \frac{1 + 0.3162}{1 - 0.3162} = 1.93 
\approx 2
\]

Hence the impedance bandwidth can also be specified by the frequency range within which \( VSWR \leq 2 \).
Prob: Calculate reflection coefficient having SWR of 1.5.

- Soln:
Prob: Calculate reflection coefficient having SWR of 1.5.

• Soln:

\[
SWR = \left| \frac{V_{\text{max}}}{V_{\text{min}}} \right| = \frac{1 + \Gamma_L}{1 - \Gamma_L}
\]

\[\therefore \quad \Gamma_L = \text{reflection coefficient} = \frac{SWR - 1}{SWR + 1}
\]

\[= \frac{1.5 - 1}{1.5 + 1} = \frac{0.5}{1.5} = \frac{1}{3}.
\]

\[\text{return loss or reflection coefficient} = -20 \log_2 \rho \quad \text{(dB)}
\]
Prob: Calculate the radiation resistance of an antenna having wavelength $5\lambda = \text{and length 25cm.}$
Prob: Calculate the radiation resistance of an antenna having wavelength $5\lambda = \text{and length 25cm.}$

\[
\text{Ans: } R_{\text{rad}} = 80\pi^2\left[\frac{dl}{\lambda}\right]^2
\]
\[
= 80\pi^2\left[\frac{25}{5}\right]^2
\]
\[
= 2000\pi^2
\]
Prob: The maximum power density radiated by a short dipole at a distance of 1 km is 60 (nW/m\(^2\)). If \(I_0 = 10\) A, find the radiation resistance.

From (9.14),

\[ S_0 = \frac{15\pi I_0^2}{R^2} \left( \frac{l}{\lambda} \right)^2 \]
Prob: The maximum power density radiated by a short dipole at a distance of 1 km is 60 (nW/m²). If $I_0 = 10$ A, find the radiation resistance.

**Solution:** From (9.14),

$$S_0 = \frac{15\pi I_0^2}{R^2} \left( \frac{l}{\lambda} \right)^2$$

and from (9.35),

$$R_{\text{rad}} = 80\pi^2 \left( \frac{l}{\lambda} \right)^2.$$

Hence,

$$R_{\text{rad}} = 80\pi^2 \left( \frac{S_0 R^2}{15\pi I_0^2} \right)$$

$$= \frac{80\pi}{15} \frac{S_0 R^2}{I_0^2}$$

$$= \frac{80\pi}{15} \times \frac{60 \times 10^{-9} \times (10^3)^2}{10^2} = 10^{-2} \Omega = 10 \text{ m\Omega}.$$
Prob: The effective area of an antenna is 9 $m^2$. What is its directivity in decibels at 3 GHz?
Prob: The effective area of an antenna is 9 m$^2$. What is its directivity in decibels at 3 GHz?

Solution: At 3 GHz,

\[
\lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m}
\]

\[
D = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi \times 9}{(0.1)^2} = 11310 = 40.53 \text{ dB}
\]
Prob: At 100 MHz, the pattern solid angle of an antenna is 1.3 sr. Find (a) the antenna directivity $D$ and (b) its effective area $A_e$. 
Prob: At 100 MHz, the pattern solid angle of an antenna is 1.3 sr. Find (a) the antenna directivity $D$ and (b) its effective area $A_e$.

Solution: At 100 MHz,

(a) \[
\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10^8} = 3 \text{ m}
\]

(b) \[
D = \frac{4\pi}{\Omega_p} = \frac{4\pi}{1.3} = 9.67.
\]

\[
A_e = \frac{\lambda^2 D}{4\pi} = \frac{3^2 \times 9.67}{4\pi} = 6.92 \text{ m}^2.
\]
Problem: The effective area of a parabolic dish antenna is approximately equal to its physical aperture. If the directivity of a dish antenna is 30 dB at 10 GHz, what is its effective area? If the frequency is increased to 30 GHz, what will be its new directivity?
Problem: The effective area of a parabolic dish antenna is approximately equal to its physical aperture. If the directivity of a dish antenna is 30 dB at 10 GHz, what is its effective area? If the frequency is increased to 30 GHz, what will be its new directivity?

Solution: At 10 GHz,

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 0.03 \text{ m.} \]

\[ A_e = \frac{\lambda^2 D}{4\pi} = \frac{0.03^2 \times 10^3}{4\pi} = 0.07 \text{ m}^2. \]

If \( f \) is increased to 30 GHz (by a factor of 3), \( \lambda \) becomes smaller by a factor of 3 and \( D \) larger by a factor of 9. Hence,

\[ D = 9 \times 10^3 = 39.44 \text{ dB}. \]
**Problem:** Determine the effective area of a half-wave dipole antenna at 100 MHz, if the wire diameter is 2 cm.
Problem: Determine the effective area of a half-wave dipole antenna at 100 MHz, if the wire diameter is 2 cm.

Solution: At $f = 100$ MHz, $\lambda = c/f = (3 \times 10^8 \text{ m/s})/(100 \times 10^6 \text{ Hz}) = 3 \text{ m}$. From Eq. (9.47), a half wave dipole has a directivity of $D = 1.64$. From Eq. (9.64), $A_e = \lambda^2 D/4\pi = (3 \text{ m})^2 \times 1.64/4\pi = 1.17 \text{ m}^2$. 
Terminology

Antenna – structure or device used to collect or radiate electromagnetic waves

Array – assembly of antenna elements with dimensions, spacing, and illumination sequence such that the fields of the individual elements combine to produce a maximum intensity in a particular direction and minimum intensities in other directions

Beamwidth – the angle between the half-power (3-dB) points of the main lobe, when referenced to the peak effective radiated power of the main lobe

Directivity – the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions

Effective area – the functional equivalent area from which an antenna directed toward the source of the received signal gathers or absorbs the energy of an incident electromagnetic wave

Efficiency – ratio of the total radiated power to the total input power

Far field – region where wavefront is considered planar

Gain – ratio of the power at the input of a loss-free isotropic antenna to the power supplied to the input of the given antenna to produce, in a given direction, the same field strength at the same distance

Isotropic – radiates equally in all directions

Main lobe – the lobe containing the maximum power

Null – a zone in which the effective radiated power is at a minimum relative to the maximum effective radiation power of the main lobe

Radiation pattern – variation of the field intensity of an antenna as an angular function with respect to the axis

Radiation resistance – resistance that, if inserted in place of the antenna, would consume that same amount of power that is radiated by the antenna

Side lobe – a lobe in any direction other than the main lobe
Friis Transmission Formula

Power density incident upon receiving antenna at a distance $R$ from an isotropic lossless transmitting antenna

$$S_{iso} = \frac{P_t}{4\pi R^2} \quad \text{(9.71)}$$

For real transmitting antenna received power density

$$S_r = G_t S_{iso} = \xi_t D_t S_{iso} = \frac{\xi_t D_t P_t}{4\pi R^2} \quad \text{(9.72)}$$

$A_t =$ Transmitting Effective Area

$A_r =$ Receiving Effective Area

$R = $ Distance between antennas

$P_t =$ Transmitter power supplied to transmitting antenna

$P_{rad} =$ Actually radiated power from transmitting antenna

$P_{int} =$ Intercepted power at receiving antenna

$\xi_t =$ Radiation efficiency of transmitting antenna

$\xi_r =$ Radiation efficiency of receiving antenna

$D_t =$ Directivity of transmitting antenna
We know effective area of any antenna is defined by

\[ A_e = \frac{\lambda^2 D}{4\pi} \]  \hspace{1cm} \text{(9.64)}

Using Eq. (9.64), Eq. (9.72) can be expressed in terms of the effective area \( A_t \)

\[ S_r = \frac{\xi_t A_t P_t}{\lambda^2 R^2} \]  \hspace{1cm} \text{(9.73)}

Power intercepted by receiving antenna = Incident power density \( S_r \times \) Effective area \( A_r \)

\[ P_{\text{int}} = S_r A_r = \frac{\xi_t A_t A_r P_t}{\lambda^2 R^2} \]  \hspace{1cm} \text{(9.74)}

The received power \( P_{\text{rec}} \) delivered to the receiver = Intercepted power \( P_{\text{int}} \times \) radiation efficiency of the receiving antenna \( \xi_r \)

\[ \frac{P_{\text{rec}}}{P_t} = \frac{\xi_t \xi_r A_t A_r}{\lambda^2 R^2} = G_t G_r \left( \frac{\lambda}{4\pi R} \right)^2 \]  \hspace{1cm} \text{(9.75)}

This relation is known as the \textit{Friis Transmission Formula} and \( \frac{P_{\text{rec}}}{P_t} \) is sometimes called the \textit{power transfer ratio}.

Where \( G_t \) and \( G_r \) are gain of transmitting and receiving antenna.
Satellite Communication System:

Problem 1:
A 6 GHz direct-broadcast TV satellite system transmits 100 W through a 2 m diameter parabolic dish antenna from a distance of approximately 40,000 km above Earth’s surface. Each TV channel occupies a bandwidth of 5 MHz. Due to electromagnetic noise picked up by the antenna as well as noise generated by the receiver electronics, a home TV receiver has a noise level given by

\[ P_n = KT_{sys}B \ (W), \]  

(9.71)

where \( T_{sys} \) [measured in kelvins (K)] is a figure of merit called the **system noise temperature** that characterizes the noise performance of the receiver–antenna combination, \( K \) is Boltzmann’s constant \([1.38 \times 10^{-23} \text{ J/K}]\), and \( B \) is the receiver bandwidth in Hz. The **signal-to-noise ratio** \( S_n \) (which should not be confused with the power density \( S \)) is defined as the ratio of \( P_{rec} \) to \( P_n \):

\[ S_n = P_{rec}/P_n \text{ (dimensionless)}. \]  

(9.72)

For a receiver with \( T_{sys} = 580 \text{ K} \),

1) what minimum diameter of a parabolic dish receiving antenna is required for high-quality TV reception with \( S_n = 40 \text{ dB} \)? The satellite and ground receiving antennas may be assumed lossless, and their effective areas may be assumed equal to their physical apertures.
Solution: The following quantities are given:

\[ P_t = 100 \text{ W}, \quad f = 6 \text{ GHz} = 6 \times 10^9 \text{ Hz}, \quad S_n = 10^4, \]

Transmit antenna diameter \( d_t = 2 \text{ m}, \)

\[ T_{sys} = 580 \text{ K}, \quad R = 40,000 \text{ km} = 4 \times 10^7 \text{ m}, \]

\[ B = 5 \text{ MHz} = 5 \times 10^6 \text{ Hz}. \]

The wavelength \( \lambda = c/f = 5 \times 10^{-2} \text{ m}, \) and the area of the transmitting satellite antenna is \( A_t = (\pi d_t^2/4) = \pi (\text{m}^2). \) From Eq. (9.71), the receiver noise power is

\[ P_n = K T_{sys} B = 1.38 \times 10^{-23} \times 580 \times 5 \times 10^6 \]

\[ = 4 \times 10^{-14} \text{ W}. \]

Using Eq. (9.69) with \( \xi_t = \xi_r = 1, \)

\[ P_{rec} = \frac{P_t A_t A_r}{\lambda^2 R^2} = \frac{100 \pi A_r}{(5 \times 10^{-2})^2 (4 \times 10^7)^2} \]

\[ = 7.85 \times 10^{-11} A_r. \]

The area of the receiving antenna, \( A_r, \) can now be determined by equating the ratio \( P_{rec}/P_n \) to \( S_n = 10^4: \)

\[ 10^4 = \frac{7.85 \times 10^{-11} A_r}{4 \times 10^{-14}}, \]

which yields the value \( A_r = 5.1 \text{ m}^2. \) The required minimum diameter is \( d_r = \sqrt{4 A_r/\pi} = 2.55 \text{ m}. \)
Problem 2: If the operating frequency of the communication system described in Problem 1 is doubled to 12 GHz, what would then be the minimum required diameter of a home receiving TV antenna?
Problem 2: If the operating frequency of the communication system described in Example 9-5 is doubled to 12 GHz, what would then be the minimum required diameter of a home receiving TV antenna?

Solution: $P_n$ remains the same, but with all other parameters remaining the same, $P_{\text{rec}}$ will increase by a factor of 4 because $P_{\text{rec}}$ is proportional to $1/\lambda^2$ and $\lambda = c/f$. This means that we can maintain $P_{\text{rec}}$ the same by reducing $A_r$ by a factor of 4, or equivalently by reducing $d_r$ by a factor of 2 down to $2.55/2 = 1.27 \text{ m.}$
Problem 3: A 3-GHz microwave link consists of two identical antennas each with a gain of 30 dB. Determine the received power, given that the transmitter output power is 1 kW and the two antennas are 10 km apart.
Problem 3: A 3-GHz microwave link consists of two identical antennas each with a gain of 30 dB. Determine the received power, given that the transmitter output power is 1 kW and the two antennas are 10 km apart.

\[ P_{\text{rec}} = P_t G_t G_r \left( \frac{\lambda}{4\pi R} \right)^2 \]

\[ P_t = 10^3 \text{ W}, \quad G_t = G_r = 10^3 \text{ (30 dB)}, \]

\[ \lambda = \frac{c}{f} = \frac{3 \times 10^8}{3 \times 10^9} = 0.1 \text{ m}, \quad R = 10^4 \text{ m}. \]

\[ P_{\text{rec}} = 10^3 \times 10^6 \left( \frac{0.1}{4\pi \times 10^4} \right)^2 = 6.33 \times 10^{-4} \text{ W}. \]
Problem 4: A 3-GHz line-of-sight microwave communication link consists of two lossless parabolic dish antennas, each 1 m in diameter. If the receive antenna requires 10 nW of receive power for good reception and the distance between the antennas is 40 km, how much power should be transmitted?
Problem 4: A 3-GHz line-of-sight microwave communication link consists of two lossless parabolic dish antennas, each 1 m in diameter. If the receive antenna requires 10 nW of receive power for good reception and the distance between the antennas is 40 km, how much power should be transmitted?

Solution: At $f = 3$ GHz, $\lambda = c/f = (3 \times 10^8 \text{ m/s})/(3 \times 10^9 \text{ Hz}) = 0.10 \text{ m}$. Solving the Friis transmission formula (Eq. (9.75)) for the transmitted power:

$$P_t = P_{\text{rec}} \frac{\lambda^2 R^2}{\zeta_t \zeta_r A_t A_r}$$

$$= 10^{-8} \frac{(0.100 \text{ m})^2 (40 \times 10^3 \text{ m})^2}{1 \times 1 \times \left(\frac{\pi}{4} (1 \text{ m})^2\right) \left(\frac{\pi}{4} (1 \text{ m})^2\right)} = 25.9 \times 10^{-2} \text{ W} = 259 \text{ mW}.$$
Problem 5: A half-wave dipole TV broadcast antenna transmits 1 kW at 50MHz. What is the power received by a home television antenna with 3-dB gain if located at a distance of 30 km?
Problem 5: A half-wave dipole TV broadcast antenna transmits 1 kW at 50 MHz. What is the power received by a home television antenna with 3-dB gain if located at a distance of 30 km?

Solution: At \( f = 50 \text{ MHz} \), \( \lambda = c/f = (3 \times 10^8 \text{ m/s})/(50 \times 10^6 \text{ Hz}) = 6 \text{ m} \), for which a half wave dipole, or larger antenna, is very reasonable to construct. Assuming the TV transmitter to have a vertical half wave dipole, its gain in the direction of the home would be \( G_t = 1.64 \). The home antenna has a gain of \( G_r = 3 \text{ dB} = 2 \). From the Friis transmission formula (Eq. (9.75)): 

\[
P_{\text{rec}} = P_t \frac{\lambda^2 G_r G_t}{(4\pi)^2 R^2} = 10^3 \frac{(6 \text{ m})^2 \times 1.64 \times 2}{(4\pi)^2 (30 \times 10^3 \text{ m})^2} = 8.3 \times 10^{-7} \text{ W}.
\]
Problem 6 A 150-MHz communication link consists of two vertical half-wave dipole antennas separated by 2 km. The antennas are lossless, the signal occupies a bandwidth of 3 MHz, the system noise temperature of the receiver is 600 K, and the desired signal-to-noise ratio is 17 dB. What transmitter power is required?
Solution: From Eq. (9.77), the receiver noise power is

\[ P_n = KT_{sys}B = 1.38 \times 10^{-23} \times 600 \times 3 \times 10^6 = 2.48 \times 10^{-14} \text{ W}. \]

For a signal to noise ratio \( S_n = 17 \text{ dB} = 50 \), the received power must be at least

\[ P_{\text{rec}} = S_nP_n = 50(2.48 \times 10^{-14} \text{ W}) = 1.24 \times 10^{-12} \text{ W}. \]

Since the two antennas are half-wave dipoles, Eq. (9.47) states \( D_t = D_r = 1.64 \), and since the antennas are both lossless, \( G_t = D_t \) and \( G_r = D_r \). Since the operating frequency is \( f = 150 \text{ MHz} \), \( \lambda = c/f = (3 \times 10^8 \text{ m/s})/(150 \times 10^6 \text{ Hz}) = 2 \text{ m} \). Solving the Friis transmission formula (Eq. (9.75)) for the transmitted power:

\[ P_t = P_{\text{rec}} \frac{(4\pi)^2 R^2}{\lambda^2 G_r G_t} = 1.24 \times 10^{-12} \frac{(4\pi)^2 (2 \times 10^3 \text{ m})^2}{(2 \text{ m})^2 (1.64)(1.64)} = 75 \text{ (\mu W)}. \]
Problem 9.22  Consider the communication system shown in Fig. 9-37 (P9.22), with all components properly matched. If $P_t = 10$ W and $f = 6$ GHz:

(a) what is the power density at the receiving antenna (assuming proper alignment of antennas)?

(b) What is the received power?

(c) If $T_{sys} = 1,000$ K and the receiver bandwidth is 20 MHz, what is the signal to noise ratio in dB?

Figure P9.22: Communication system of Problem 9.22.
Solution:

(a) \( G_t = 20 \text{ dB} = 100, \quad G_r = 23 \text{ dB} = 200, \) and \( \lambda = c/f = 5 \text{ cm}. \) From Eq. (9.72),
\[
S_r = G_t \frac{P_t}{4\pi R^2} = \frac{10^2 \times 10}{4\pi \times (2 \times 10^4)^2} = 2 \times 10^{-7} \quad (\text{W/m}^2).
\]

(b)
\[
P_{\text{rec}} = P_t G_t G_r \left( \frac{\lambda}{4\pi R} \right)^2 = 10 \times 100 \times 200 \times \left( \frac{5 \times 10^{-2}}{4\pi \times 2 \times 10^4} \right)^2 = 7.92 \times 10^{-9} \text{ W}.
\]

(c)
\[
P_n = K T_{\text{sys}} B = 1.38 \times 10^{-23} \times 10^3 \times 2 \times 10^7 = 2.76 \times 10^{-13} \text{ W},
\]
\[
S_n = \frac{P_{\text{rec}}}{P_n} = \frac{7.92 \times 10^{-9}}{2.76 \times 10^{-13}} = 2.87 \times 10^4 = 44.6 \text{ dB}.
\]
Polarization

• The polarization of an antenna is the orientation of the electric field with respect to the Earth's surface and is determined by the physical structure of the antenna and by its orientation.

• Radio waves from a vertical antenna will usually be vertically polarized.

• Radio waves from a horizontal antenna are usually horizontally polarized.
Direction of Propagation
Horizontally polarized directional yagi antenna

Vertically polarized omnidirectional dipole antenna
Linearly polarized

Circularly polarized

Elliptically polarized

See animation “Polarization of a Plane Wave - 2D View”

See animation “Polarization of a Plane Wave - 3D View”
Polarization of Plane Waves

(a) Linear polarization

A plane wave is linearly polarized at a fixed observation point if the tip of the electric-field vector at that point moves along the same straight line at every instant of time.

(b) Circular Polarization

A plane wave is circularly polarized at a fixed observation point if the tip of the electric-field vector at that point traces out a circle as a function of time.
Circular polarization can be either right-handed or left-handed corresponding to the electric-field vector rotating clockwise (right-handed) or anti-clockwise (left-handed).

(c) Elliptical Polarization

A plane wave is elliptically polarized at a fixed observation point if the tip of the electric-field vector at that point traces out an ellipse as a function of time. Elliptically polarization can be either right-handed or left-handed corresponding to the electric-field vector rotating clockwise (right-handed) or anti-clockwise (left-handed).
Axial Ratio

The polarization state of an EM wave can also be indicated by another two parameters: Axial Ratio (AR) and the tilt angle (τ). AR is a common measure for antenna polarization. It definition is:

\[ \text{AR} = \frac{\text{OA}}{\text{OB}}, \quad 1 \leq \text{AR} \leq \infty, \quad \text{or} \quad 0 \text{ dB} \leq \text{AR} \leq \infty \text{dB} \]

where OA and OB are the major and minor axes of the polarization ellipse, respectively. The tilt angle τ is the angle subtended by the major axis of the polarization ellipse and the horizontal axis.
\[ \tau = \text{tilt angle} \]
\[ 0 \leq \tau \leq 180^\circ \]
For example:
\[
AR = 1, \quad \Rightarrow \text{circular polarization}
\]
\[
1 < AR < \infty, \quad \Rightarrow \text{elliptical polarization}
\]
\[
AR = \infty, \quad \Rightarrow \text{linear polarization}
\]

AR can be measured experimentally!

Very often, we use the **AR bandwidth** and the **AR beamwidth** to characterize the polarization of an antenna. The AR bandwidth is the frequency bandwidth in which the AR of an antenna changes less than 3 dB from its minimum value. The AR beamwidth is the angle span over which the AR of an antenna changes less than 3 dB from its minimum value.
Test antenna (receiving)

Fast-rotating dipole antenna (transmitting)

Radiation pattern with a rotating linear source

AR at $\theta$ in dB scale

3 dB AR beamwidth
Axial ratio (dB)

3dB

AR bandwidth

Frequency
Thank You
\[ \Gamma = \frac{VSWR - 1}{VSWR + 1} \quad RL = -20 \log \left( \frac{VSWR - 1}{VSWR + 1} \right) \quad ML = -10 \log \left( 1 - \left( \frac{VSWR - 1}{VSWR + 1} \right)^2 \right) \]

\[ VSWR = \frac{1 + \Gamma}{1 - \Gamma} \quad RL = -20 \log (\Gamma) \quad ML = -10 \log (1 - \Gamma^2) \]

\[ \Gamma = 10^{\frac{-RL}{20}} \quad VSWR = \frac{1 + 10^{\frac{-RL}{20}}}{1 - 10^{\frac{-RL}{20}}} \quad ML = -10 \log \left[ 1 - \left( 10^{\frac{-RL}{20}} \right)^2 \right] \]
Friis' transmission formula

At a fixed distance $R$ from the transmitting antenna, the power intercepted by the receiving antenna with effective aperture $A_r$ is

$$P_i = S_r A_r = \frac{P_t}{4 \pi R^2} G_t A_r$$

where $S_r$ is the received power density (W/m$^2$), and $G_t$ is the peak gain of the transmitting antenna.

Fig. 3.6  Transmitter-receiver configuration.